Clustering Some Problems from the Erdős Problems Website

Findings from the present chat

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Purpose and convention

This note summarizes the classification suggestions made in the chat about the problem collection at `https://www.erdosproblems.com/`. The groupings below are not meant as official taxonomy. They are meant as a useful working taxonomy for navigating related problems.

I use the term *fine cluster* for a narrow mathematical neighbourhood: problems in the same fine cluster should share not only broad area, but also a similar main object, extremal quantity, and expected techniques. Thus two problems may be in the same broad family but in different fine clusters.

A proposed coarse clustering into 15 groups

A grouping into fewer than 10 clusters would be too coarse. In particular, the site's broad tags such as number theory, graph theory, Ramsey theory, geometry, additive combinatorics, and analysis each contain several distinct mathematical cultures. A useful compromise is the following list of 15 coarse clusters.

	Cluster	Typical content
1	Additive and combinatorial number theory	Sumsets, subset sums, additive bases, Sidon-type questions, restricted sums.
2	Prime numbers and analytic number theory	Prime gaps, primes in patterns, primes plus powers, distributional prime questions.
3	Arithmetic progressions and density phenomena	Roth/Szemeredi-type questions, dense sets, reciprocal-sum conditions forcing structure.
4	Covering systems and congruence constructions	Covering congruences, residue classes, modular obstructions, finite verifiable systems.
5	Divisibility, multiplicative structure, and special integers	Divisors, primitive sets, factorials, binomial coefficients, powerful numbers, squares and higher powers.
6	Unit fractions, reciprocal sums, irrationality, and Diophantine approximation	Egyptian fractions, harmonic sums, irrationality of series, approximation constants.
7	Extremal graph theory and hypergraphs	Turan problems, extremal edge counts, forbidden subgraphs, hypergraph extremal questions.
8	Ramsey, colouring, and chromatic problems	Ramsey numbers, graph colourings, chromatic number, unavoidable monochromatic structures.
9	Graph structure, cycles, and special graph classes	Cycles, planar graphs, graph diameter and connectivity questions, structural graph problems.
10	Discrete and combinatorial geometry	Unit distances, repeated distances, convex sets, point configurations, geometric incidence-style questions.

	Cluster	Typical content
11	Analysis, approximation, and polynomials	Interpolation, polynomial inequalities, analytic estimates, approximation theory.
12	Probability, discrepancy, and random methods	Probabilistic constructions, discrepancy bounds, random colourings, concentration-style arguments.
13	Set theory, topology, and infinite combinatorics	Infinite sets, independence phenomena, topological combinatorics, set-theoretic extremal questions.
14	Iterated functions, sequences, and representations	Dynamical or iterated maps, base representations, recurrence-like or sequence problems.
15	Algebraic and group-theoretic problems	Group actions, algebraic structures, algebraic extremal questions.

The clusters should not be made disjoint. A natural implementation would assign each problem one primary cluster and possibly one or two secondary clusters.

Problem #52 versus problem #90

The two problems are not in the same *fine cluster*.

Problem #52 is the *sum-product problem.* For a finite set of integers A, it asks whether for every $\epsilon > 0$ one has

$$\max\bigl(|A+A|, |AA|\bigr) \gg_\epsilon |A|^{2-\epsilon}.$$

Its natural fine cluster is additive–multiplicative expansion of finite arithmetic sets.

Problem #90 is the planar *unit distance problem.* It asks whether every set of n distinct points in $\mathbb{R}^2$ has at most

$$n^{1+O(1/\log\log n)}$$

pairs at distance 1. Its natural fine cluster is unit-distance and repeated-distance extremal geometry.

Thus the two problems do share a broad flavour: both ask for sharp extremal exponents for structured finite configurations. But this is only a broad meta-family. Their fine clusters are different:

$$\begin{aligned} &\#52: && \text{additive/multiplicative expansion, sum-product phenomena,} \\ &\#90: && \text{discrete geometry, unit distances, distance graphs.} \end{aligned}$$

Fine cluster around problem #90

For problem #90, I would use the following fine cluster:

Unit-distance / repeated-distance extremal geometry in Euclidean point sets.

The strict core of the cluster is:

Problem	Reason for inclusion
#90	The central planar unit-distance problem: maximize the number of pairs at distance 1 among n planar points.
#92	A stronger local version: each point should have many equidistant neighbours; the site describes it as a stronger form of the unit-distance conjecture.
#96	Unit distances in convex n-gons: a convex-position version of the unit-distance question.
#605	Equal or unit distances among points on a sphere; explicitly adjacent to #90.
#668	Extremal-configuration version: asks about the number of incongruent n-point sets maximizing the number of unit distances.
#956	Unit distances between disjoint convex translates; the page relates its function $h(n)$ to the classical unit-distance maximum.
#1085	Higher-dimensional version of the unit-distance problem; the case $d = 2$ is precisely the planar unit-distance problem.

There are also close but not core neighbours:

- #1084, the contact-number problem: points are required to be mutually at distance at least 1. This is very close geometrically, but has a packing/contact-number flavour rather than the unrestricted repeated-distance flavour of #90.
- #223, the diameter-one version: maximize the number of distance-1 pairs among n points of diameter 1. This is close to the contact and diameter side of the same geometric family.
- #705, a unit-distance graph colouring problem with girth restrictions. It uses unit-distance graphs, but its main quantity is chromatic/graph-theoretic rather than an extremal number of distance pairs.

I would not put the distinct-distances problem into the same fine cluster as #90. It lies in the same broad Erdős distance-problem family, but the extremal direction is different: distinct distances ask for many different distances, while #90 asks how often one fixed distance can repeat.

Fine cluster around problem #52

For problem #52, I would use the following fine cluster:

Sum-product and additive–multiplicative expansion of finite integer sets.

The strict core is small:

Problem	Reason for inclusion
#52	The central Erdős–Szemeredi sum-product problem for finite integer sets.
#53	Higher/variant form: many integers obtainable as a sum or product of distinct elements of A; the page explicitly points back to #52.

Problem	Reason for inclusion
#808	Graph-restricted sum-product version, with sums and products taken along the edges of a graph G on A; this is described as a strengthening of #52.
#818	Small-sumset special case: if $\lvert A+A\rvert \ll \lvert A\rvert$, prove that $\lvert AA\rvert$ is almost quadratic, up to logarithmic loss; the page explicitly links it to #52.

Thus my strict fine cluster for #52 is

$$\boxed{\#52,\ \#53,\ \#808,\ \#818.}$$

I would not include all additive-combinatorics problems here. Sumset-only questions, additive-basis questions, Sidon-set problems, arithmetic-progression questions, and additive-complement problems belong to broader additive combinatorics, but not to this fine cluster unless multiplication/product structure is genuinely central.

Summary of the two fine clusters

The two important fine clusters from this discussion are therefore:

$$\begin{aligned}\textbf{Around } \#\textbf{90:}\quad & \{\#90, \#92, \#96, \#605, \#668, \#956, \#1085\},\\ \textbf{Around } \#\textbf{52:}\quad & \{\#52, \#53, \#808, \#818\}.\end{aligned}$$

The first is a discrete-geometry cluster about repeated equal distances. The second is an additive-combinatorics cluster about simultaneous additive and multiplicative growth. The common umbrella is "extremal exponent problems for structured finite configurations", but that umbrella is too broad for a fine cluster.

Source pages consulted

The following Erdős Problems pages were used as source pages for the problem descriptions and cross-links:

- Problem #52: `https://www.erdosproblems.com/52`
- Problem #53: `https://www.erdosproblems.com/53`
- Problem #90: `https://www.erdosproblems.com/90`
- Problem #92: `https://www.erdosproblems.com/92`
- Problem #96: `https://www.erdosproblems.com/96`
- Problem #223: `https://www.erdosproblems.com/223`
- Problem #605: `https://www.erdosproblems.com/605`
- Problem #668: `https://www.erdosproblems.com/668`
- Problem #705: `https://www.erdosproblems.com/705`

- Problem #808: `https://www.erdosproblems.com/808`
- Problem #818: `https://www.erdosproblems.com/818`
- Problem #956: `https://www.erdosproblems.com/956`
- Problem #1084: `https://www.erdosproblems.com/1084`
- Problem #1085: `https://www.erdosproblems.com/1085`