Dynamic Strategies for Mission Launches and Secretary Problems

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* Basic Model, Recursions, Results
* Races for First
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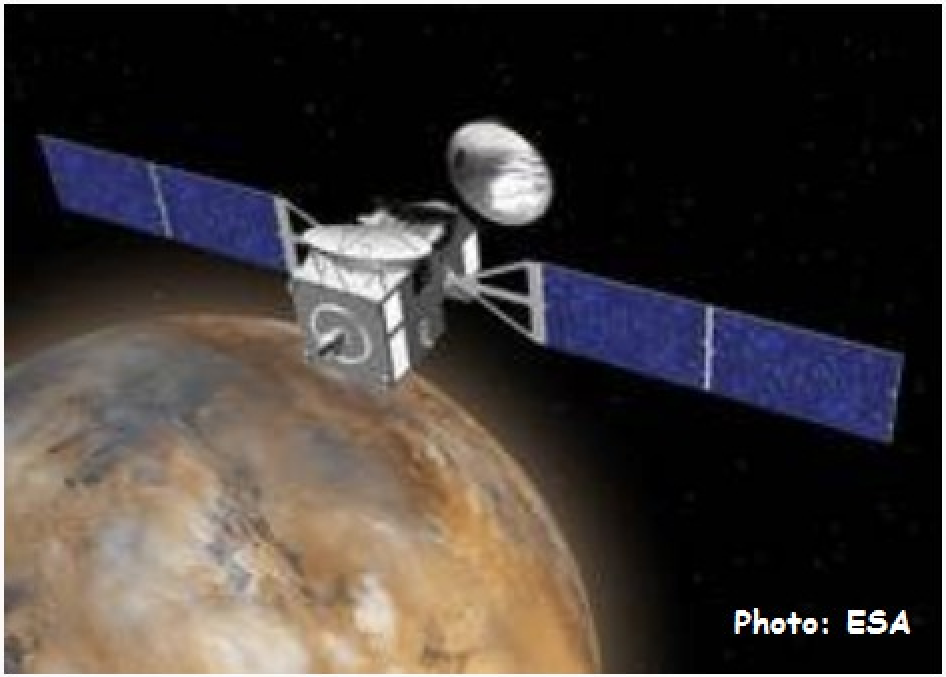
Launch Scenario: 13 days, one slot per day

ExoMars Launch March 14 - 26, 2016

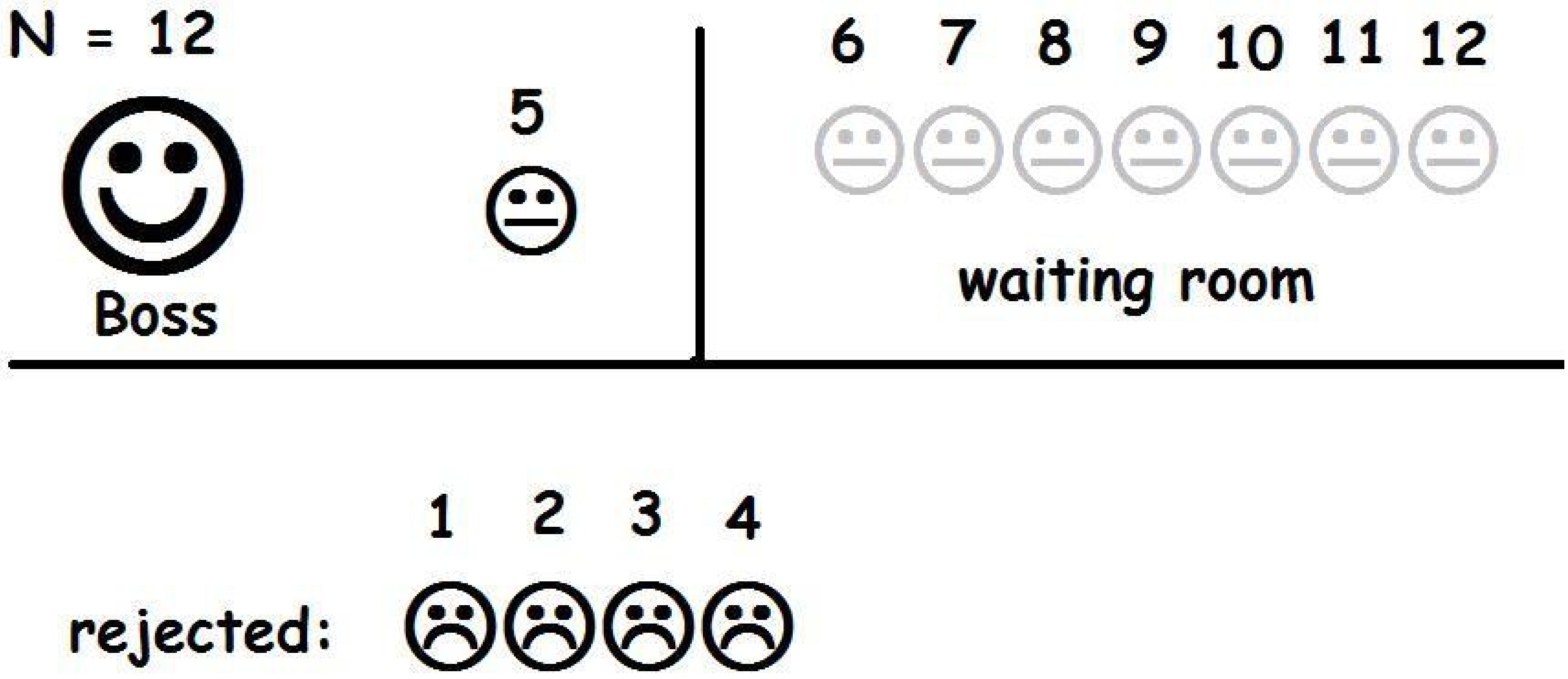


ExoMars Trace Gas Orbiter with *Schiaparelli* lander

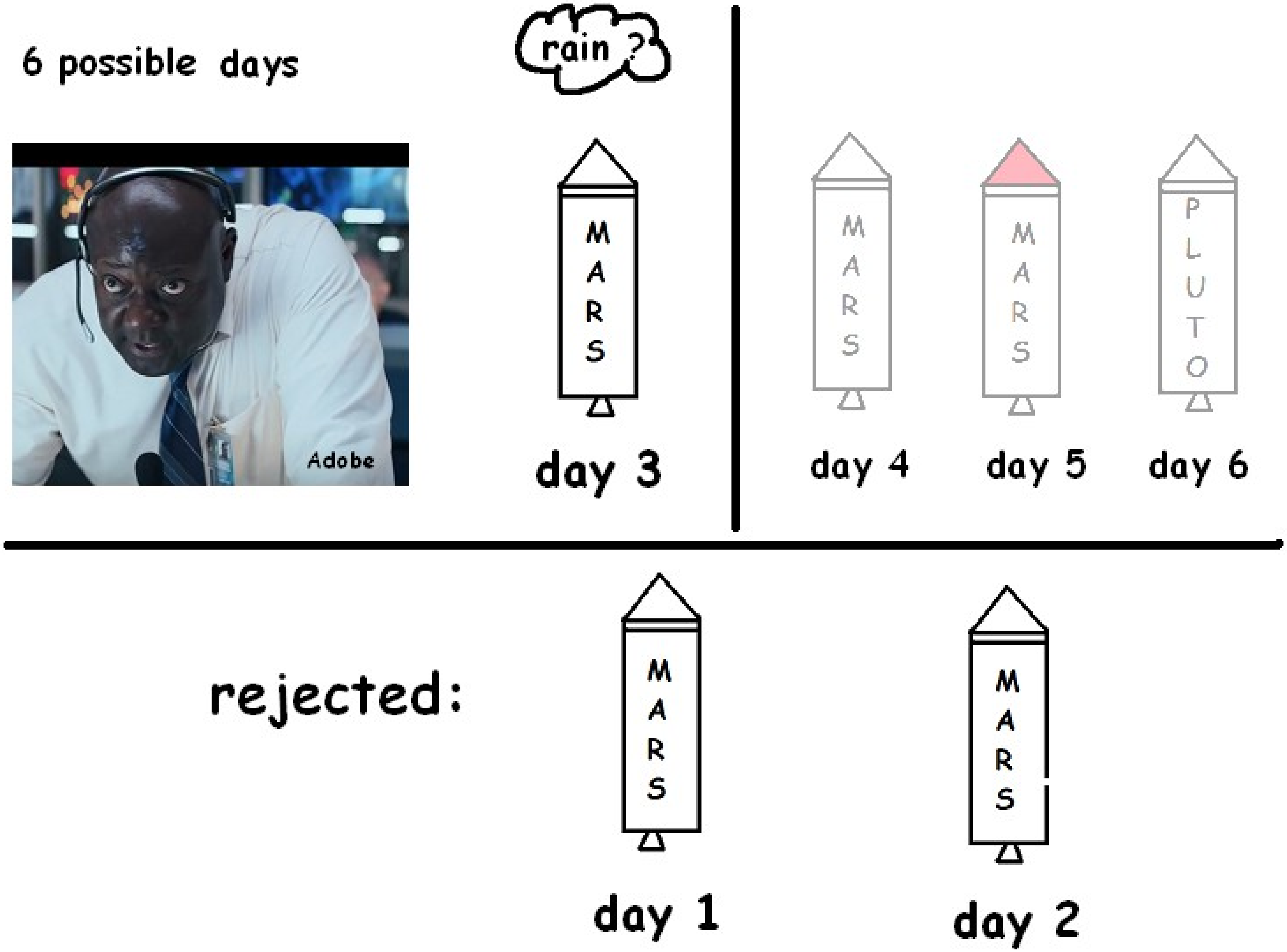
November 15, 1988: Soviet Space Glider Buran

Was it the last chance for a launch ?

Secretary Problem



Launch as a Secretary Problem



A Basic Stochastic Model

Only one rocket available.
N possible launch days;
Launch Director knows number N in advance.

Each day i has some chance x(i) of success;

$$0.00 < x(i) < 1.00$$

x(i) = 0.00 = sure failure; x(i) = 1.00 = sure success

His/her decision: launch on day i or no launch ?

(1) $x(i)$ becomes known only directly ahead of date i.

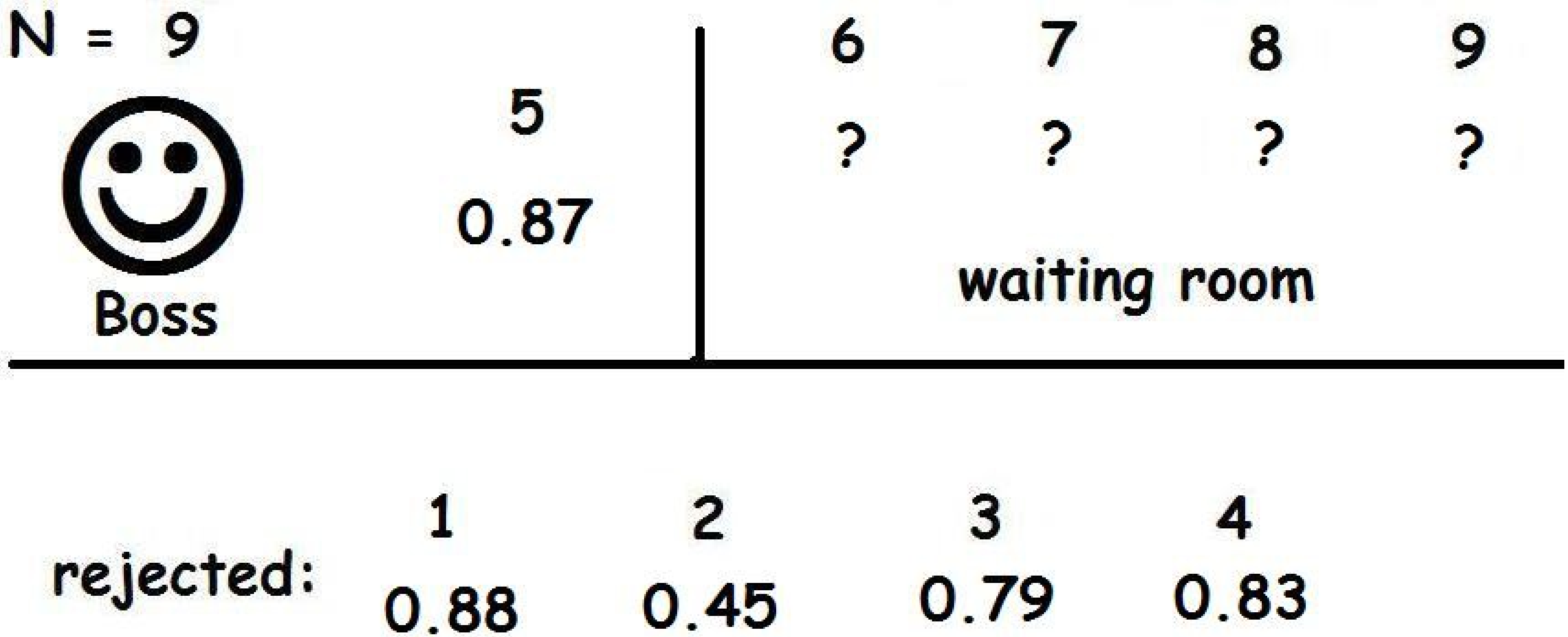
**(2) The $x(i)$ are random numbers,
independent of each other, identically distributed.**

**(3) Very special: The $x(i)$ are uniformly distributed
in the interval [0,1].**

**(4) The Director wants to maximize the probability
of a successful launch.**

**(5) The launch attempt has to take place on one
of the days. No postponement to other years allowed.**

Example with Numbers



OPTIMAL STATIC STRATEGY

Threshold T: Launch on current day, if x( ) > T .

Optimal threshold is $T = 1 - \log(N) / N$;

leads to success rate of about $1 - \log(N) / 2N$.

DYNAMIC STRATEGY

Make threshold $T = T(i)$ dependent on the day i

The more days are left, the higher $T(i)$.

The optimal thresholds can be computed
in backward order by Bellman recursions.

Let $E(i)$ be the expected score of the optimal strategy when still i candidates are there.

$E(0) = 0.0$

$E(1) = 0.5$

$T(1) = 0.0$:
you have to take the last candidate, and it has expected value 0.5

Let x(2) be given, the value of the second to last chance.

For $x(2) < 0.5$ it is optimal to postpone the launch to the last day

For $x(2) > 0.5$ it is optimal to try launch on day $i = 2$.

Hence, $T(2) = 0.5$

$$E(2) = \text{Prob}[x(2) < 0.5] * E(1) + \text{Prob}[x(2) > 0.5] * \text{Exp}[x(2) \mid x(2) > 0.5]$$

$$= 0.5 * 0.5 + 0.5 * 0.75 = 0.625$$

General Step $i > 0$

$$E(i) = Prob[x(i) < E(i-1)] * E(i-1) + Prob[x(i) > E(i-1)] * C[E(i-1)]$$

where $C[E(i-1)]$ is the conditional expected value of $x(i)$ for $x(i) > E(i-1)$.

$$C[E(i-1)] = [E(i-1) + 1] / 2.$$

This leads to

$$E(i) = E(i-1)*E(i-1) + [1 - E(i-1)]*[E(i-1) + 1] / 2$$
$$= 0.5 + 0.5 * E(i-1)*E(i-1)$$

i	E(i)
0	0.000
1	0.500
2	0.625
3	0.695
4	0.742
5	0.775
6	0.800
7	0.820
8	0.836
9	0.850
10	0.861
11	0.871
12	0.879
13	0.886

E(i)-values are monotonically increasing and converging to 1, for i to infinity.

T(i) = E(i-1) for all i.

TEAMS IN COMPETITION

PART-TIME SCIENTISTS

TEAM INDUS

ASTROBOTIC

Google LUNAR XPRIZE

TEAM SPACEIL

HAKUTO

MOON EXPRESS

Land a rover on the moon before the end of 2017.

First	**performing private team gets**	**$US 20 Millions.**
Second	**performing private team gets**	**$US 5 Millions.**

6+ serious competitors!

* * * * * * * * * *

Abstract model with two teams A and B; no second prize.
Launch opportunities in alternating order.

For odd values of i, team A has a launch opportunity.
For even values of i, it is B's turn.

For each i, chance level x(i) is defined like before.

Also the five conditions (1) to (5) are assumed to hold.

When one of the teams had an unsuccessful launch at some day i, the other team gets **all** remaining launch opportunities i-1, i-2, …, 1.

Values E(i) like before in the 1-team model.

a(i) is the expected score of team A, when still i launch days are available.

b(i) is the corresponding expected score for team B.

Of course, starting values are **$a(0) = b(0) = 0.0$** .
Recursions (from i to i+1) depend on the parity of i.

For even i let a(i), b(i), E(i) be known.

Now given x(i+1), team A should try the launch if $x(i+1) > a(i)$. This means $T(i+1) = a(i)$ and

$$a(i+1) = a(i)*a(i) + 0.5*[1 - a(i)*a(i)],$$
$$b(i+1) = a(i)*b(i) + 0.5*[1 - a(i)]*[1 - a(i)]*E(i).$$

Analogously, for odd values of i we get

$$b(i+1) = b(i)*b(i) + 0.5*[1 - b(i)*b(i)],$$
$$a(i+1) = b(i)*a(i) + 0.5*[1 - b(i)]*[1 - b(i)]*E(i).$$

i	$T_2(i)$	$T(i)$
1	0.000	0.000
2	0.000	0.500
3	0.250	0.625
4	0.301	0.695
5	0.330	0.742
6	0.346	0.775
7	0.358	0.800
8	0.365	0.820
9	0.371	0.836
10	0.376	0.850
11	0.379	0.861
12	0.382	0.871
13	0.385	0.879

Observation: For very large i launch shall be tried, if
x(i) > square-root (2) – 1 = 0.414

* *

The same model, but with second prize 0.25 (25 percent)

Result: For very large i, launch should be tried, if
x(i) > [sq(129) – 7] / 8 = 0.545

Main Insights

* Much higher risks are optimal, if there is a competitor.

* Second prize does not change much.

* Even higher risks are optimal in case of more than two teams   (recursions not shown here).

Near the end of 2017 there is a good chance to see several (non-successful) Lunar X missions if teams "play" optimally in a game-theoretic sense.

Historical Space Race Situations

US 1957/1958: Air Force vs. Army (first US-satellite)

USA vs USSR 1968: Men around the Moon

Zond 5 September 1968

RELATED SECRETARY SCENARIOS

* **Restart from Mars (during long global dust storms ?!)**

* **Tethered Flyby at some Small Asteroid (harpooning)**

* **Launch Insurances (MunichRe):**
  "Launch Insurance Packages" for the space flight market.

Two Key Problems within our Models

* **In reality, the chance values x(i) for different launch days are <u>not stochastically independent</u>. Models with dependence structures are much more complicated. (Analysis is underway by C. Pressel and T. Hetz)**

* **Finding distributions for the true x(i)-values is non-trivial.**

THANKS TO

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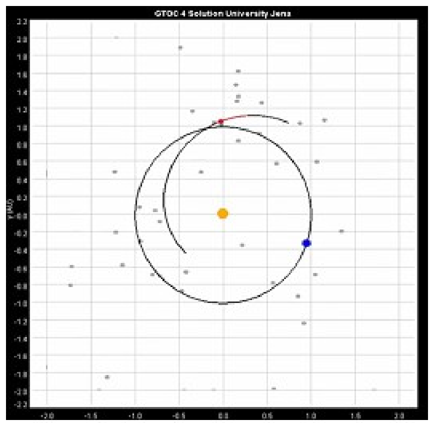

GTOC-4 with 46 Flybys